OCR
Oxford Cambridge and RSA

Friday 20 May 2016 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 6 3 8 6 6 9 5 4 3 2 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 8 & -2 \\ p & 1 \end{pmatrix}$, where $p \neq -4$.

(i) Find the inverse of **M** in terms of p. **[2]**

(ii)

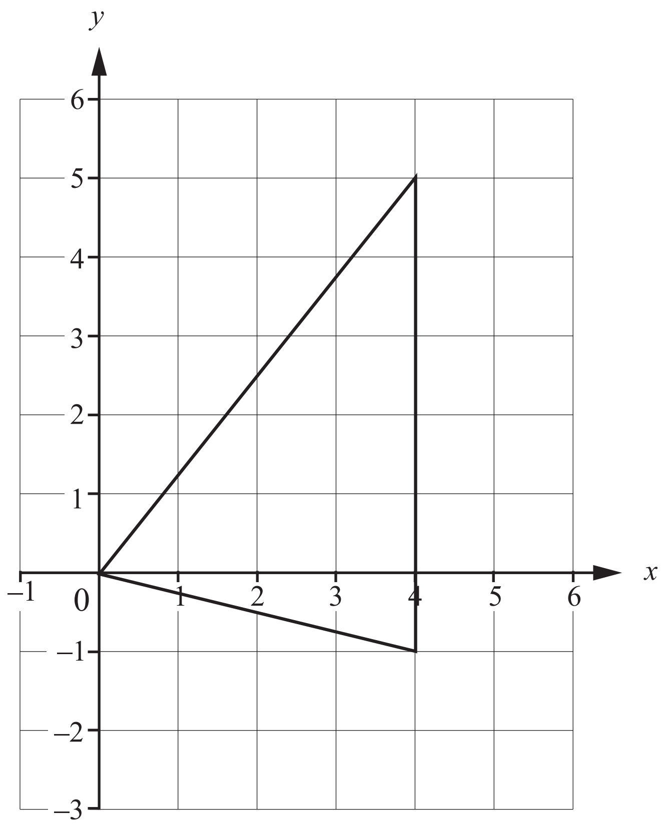

Fig. 1

The triangle shown in Fig. 1 undergoes the transformation represented by the matrix $\begin{pmatrix} 8 & -2 \\ 3 & 1 \end{pmatrix}$. Find the area of the image of the triangle following this transformation. **[2]**

2 The complex number z_1 is $2 - 5\text{j}$ and the complex number z_2 is $(a-1) + (2-b)\text{j}$, where a and b are real.

(i) Express $\frac{z_1^*}{z_1}$ in the form $x + y\text{j}$, giving x and y in exact form. You must show clearly how you obtain your answer. **[4]**

(ii) Given that $\frac{z_1^*}{z_1} = z_2$, find the exact values of a and b. **[2]**

3 You are given that $\mathbf{A} = \begin{pmatrix} \lambda & 6 & -4 \\ 2 & 5 & -1 \\ -1 & 4 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -19 & 34 & -14 \\ 5 & -5 & 5 \\ -13 & 18 & -3 \end{pmatrix}$ and $\mathbf{AB} = \mu\mathbf{I}$, where $\mathbf{I}$ is the 3×3 identity matrix.

(i) Find the values of λ and μ. **[4]**

(ii) Hence find $\mathbf{B}^{-1}$. **[2]**

4 **(i)** Use standard series to show that

$$\sum_{r=1}^{n} r^2(2r-p) = \frac{1}{6}n(n+1)(3n^2+(3-2p)n-p),$$

where p is a constant. **[4]**

(ii) Given that the coefficients of n^3 and n^4 in the expression for $\sum_{r=1}^{n} r^2(2r-p)$ are equal, find the value of p. **[2]**

5 The loci C_1 and C_2 are given by $|z+3-4\mathrm{j}| = 5$ and $\arg(z+3-6\mathrm{j}) = \frac{1}{2}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[5]**

(ii) Write down the complex number represented by the point of intersection of C_1 and C_2. **[1]**

(iii) Indicate, by shading on your sketch, the region satisfying

$$|z+3-4\mathrm{j}| \geqslant 5 \quad \text{and} \quad \frac{1}{2}\pi \leqslant \arg(z+3-6\mathrm{j}) \leqslant \frac{3}{4}\pi.$$ **[2]**

6 A sequence is defined by $u_1 = 8$ and $u_{n+1} = 3u_n + 2n + 5$. Prove by induction that $u_n = 4(3^n) - n - 3$. **[6]**

Section B (36 marks)

7 The function $\mathrm{f}(z) = 2z^4 - 9z^3 + Az^2 + Bz - 26$ has real coefficients. The equation $\mathrm{f}(z) = 0$ has two real roots, α and β, where $\alpha > \beta$, and two complex roots, γ and δ, where $\gamma = 3 + 2\mathrm{j}$.

(i) Show that $\alpha + \beta = -\frac{3}{2}$ and find the value of $\alpha\beta$. **[5]**

(ii) Hence find the two real roots α and β. **[3]**

(iii) Find the values of A and B. **[3]**

(iv) Write down the roots of the equation $\mathrm{f}\left(\frac{w}{\mathrm{j}}\right) = 0$. **[2]**

8 A curve has equation $y = \dfrac{3x^2 - 9}{x^2 + 3x - 4}$.

(i) Find the equations of the two vertical asymptotes and the one horizontal asymptote of this curve. **[3]**

(ii) State, with justification, how the curve approaches the horizontal asymptote for large positive and large negative values of x. **[3]**

(iii) Sketch the curve. **[3]**

(iv) Solve the inequality $\dfrac{3x^2 - 9}{x^2 + 3x - 4} \geqslant 0$. **[3]**

9 You are given that $\dfrac{3}{4(2r-1)} - \dfrac{1}{2r+1} + \dfrac{1}{4(2r+3)} = \dfrac{2r+5}{(2r-1)(2r+1)(2r+3)}$.

(i) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{2r+5}{(2r-1)(2r+1)(2r+3)} = \frac{2}{3} - \frac{3}{4(2n+1)} + \frac{1}{4(2n+3)}.$$ **[6]**

(ii) Write down the limit to which $\sum_{r=1}^{n} \dfrac{2r+5}{(2r-1)(2r+1)(2r+3)}$ converges as n tends to infinity. **[1]**

(iii) Find the sum of the finite series

$$\frac{45}{39 \times 41 \times 43} + \frac{47}{41 \times 43 \times 45} + \frac{49}{43 \times 45 \times 47} + \ldots + \frac{105}{99 \times 101 \times 103},$$

giving your answer to 3 significant figures. **[4]**

END OF QUESTION PAPER

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Friday 20 May 2016 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
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- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1 (i)	
1 (ii)	

2 (i)	
2 (ii)	

3 (i)	
3 (ii)	

4 (i)	
4 (ii)	

5 (i) & (iii)	**There is spare blank space on page 16. If you wish to offer a second attempt, then you must cross through the attempt on this page.**
5 (ii)	

6	
	(answer space continued on next page)

6	(continued)

Section B (36 marks)

7 (i)

7 (ii)	
7 (iii)	

7 (iv)	
8 (i)	
8 (ii)	

8 (iii)	

8 (iv)

9 (i)

9 (ii)	
9 (iii)	

This is spare blank space for Question 5 parts (i) and (iii). Only write on this page if you want to offer a second attempt. If you do so then you must cross through the first attempt on page 6.

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GCE

Mathematics (MEI)

Unit **4755**: Further Concepts for Advanced Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2016

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
	Highlighting

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

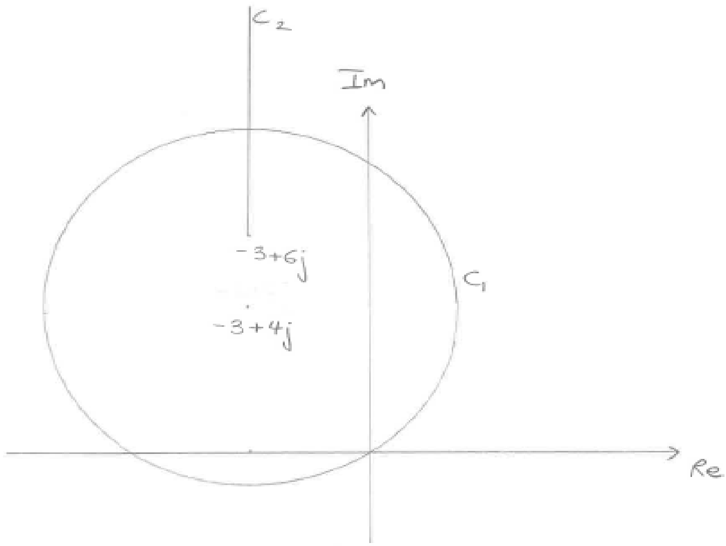
NB Follow these maths-specific instructions rather than those in the assessor handbook.

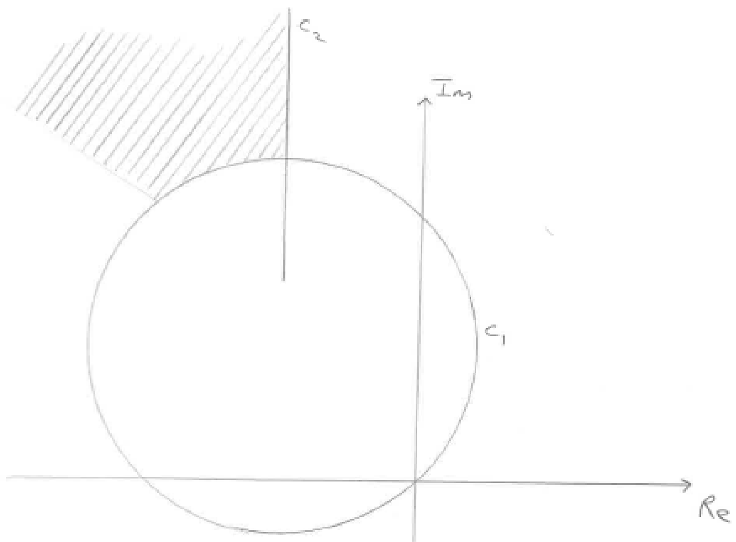
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	$\mathbf{M}^{-1}=\frac{1}{8+2p}\begin{pmatrix}1 & 2\\ -p & 8\end{pmatrix}$	B1 B1 **[2]**	Correct determinant correctly used Correct re-arrangements of elements
1	(ii)	Area of image = triangle area × (8 + 2p) = $12(8+2p)$ = 168 (square units)	M1 A1 **[2]**	Multiplying an area by their determinant with $p=3$ (accept 8 $\underline{+}$ 2 × 3 only) **Or** new coords and **valid** method to area cao
2	(i)	$z_1^*=2+5\text{j}$ $\frac{z_1^*}{z_1}=\frac{(2+5\text{j})(2+5\text{j})}{(2-5\text{j})(2+5\text{j})}=-\frac{21}{29}+\frac{20}{29}\text{j}$	B1 M1 A1 A1 **[4]**	 Correct use of conjugate 29 in denominator All correct
2	(ii)	$a-1=-\frac{21}{29}$ **and** $2-b=\frac{20}{29}$ $a=\frac{8}{29}$, $b=\frac{38}{29}$	M1 A1 **[2]**	Equating real **and** imaginary parts Both, ft their (i)
3	(i)	Either $\mu=(2)(34)+(5)(-5)+(-1)(18)=25$ Or $\mu=(-1)(-14)+(4)(5)+(3)(-3)$ $-19\lambda+(6)(5)+(-4)(-13)=25$ Or $34\lambda-102=0$ Or $-14\lambda+42=0$	M1 M1 A1 A1 **[4]**	Multiplying a row of **A** by a column of **B** to find λ or μ Multiplying another row of **A** by a column of **B** to find the other unknown μ = 25 cao λ = 3 cao

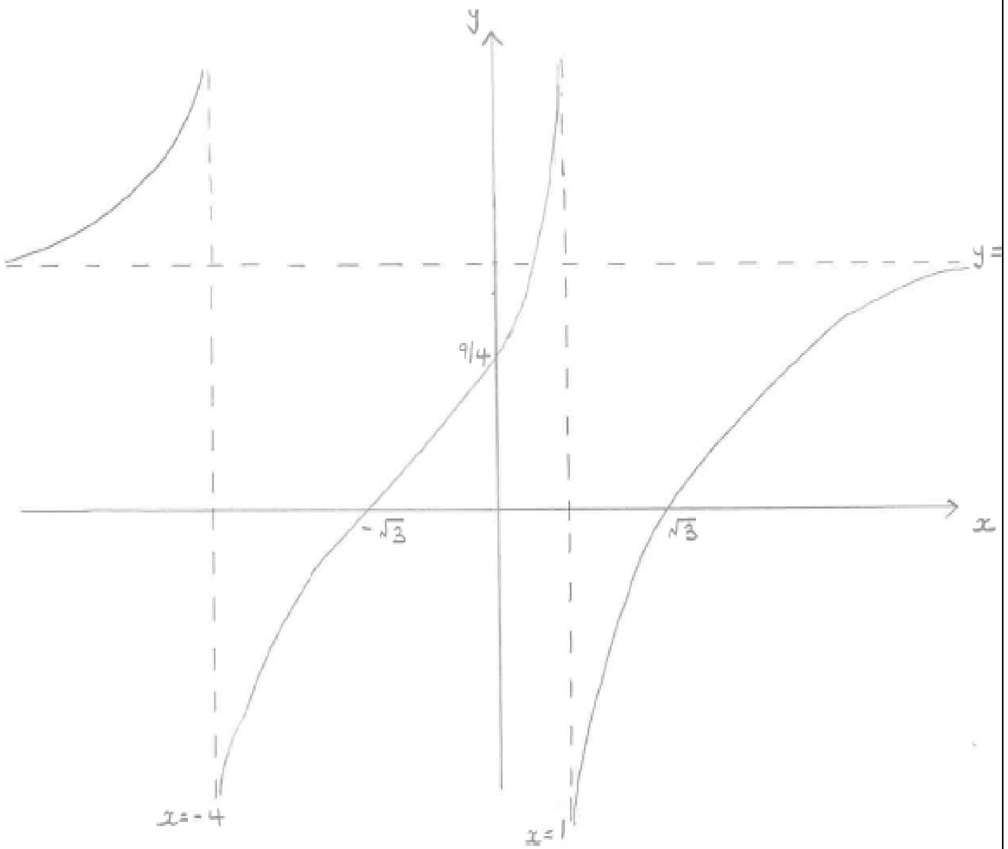
Question		Answer	Marks	Guidance
3	(ii)		M1	$\mathbf{B}^{-1} = \frac{1}{\mu}\mathbf{A}$ with their λ and **their** μ
		$\mathbf{B}^{-1} = \frac{1}{25}\begin{pmatrix} 3 & 6 & -4 \\ 2 & 5 & -1 \\ -1 & 4 & 3 \end{pmatrix}$	B1	cao
			[2]	NB μ = 1/25 in (i) then 25**A** earns M1 B0 whereas 1/25 **A** earns M0 B1
4	(i)	$\sum_{r=1}^{n} r^2(2r-p) = 2\sum_{r=1}^{n} r^3 - p\sum_{r=1}^{n} r^2$	M1*	Splitting into sums $a\sum r^3 \pm b\sum r^2$
		$= \frac{1}{2}n^2(n+1)^2 - \frac{1}{6}pn(n+1)(2n+1)$ o.e.	A1	Use of standard results in terms of n
		$= \frac{1}{6}n(n+1)(3n(n+1) - p(2n+1))$	M1dep	Attempt to factorise with $n(n+1)$ If from quartic in n all steps justified, otherwise M0
		$= \frac{1}{6}n(n+1)(3n^2 + (3-2p)n - p)$	A1 **[4]**	AG
4	(ii)	$3 = 6 - 2p$	M1	Equating their coefficients of n^3 and n^4
		$p = \frac{3}{2}$	A1 **[2]**	

Question		Answer	Marks	Guidance
5	(i)		B1 B1 B1 B1 B1 **[5]**	Accept un-numbered evenly spaced marks on axes to show scale Circle Centre $-3+4\text{j}$ Radius = 5 (check that the circle passes through O or any valid point or explicitly shown) allow for centre $\mp3\pm4j$ Half line $\frac{1}{2}\pi$ from $\mp3\pm6j$ Fully correct
5	(ii)	$-3+9\text{j}$	B1 **[1]**	Not (-3, 9j) nor (-3, 9)

Question		Answer	Marks	Guidance
5	iii		B1 B1 **[2]**	Region outside circle, must have a border with the circumference Correct region shown, boundary at $\frac{3\pi}{4}$ reasonably well drawn
6		When $n=1$, $u_n = 4(3)^1 - 1 - 3 = 8$, (so true for n = 1) Assume $u_k = 4(3)^k - k - 3$	B1 E1	Showing use of $u_n = 4(3)^n - n - 3$ Assuming true for $n = k$ Allow "let n = k and (result)" or "If n = k and (result)" Do not allow "n=k" or "let n=k" without the result quoted

Question		Answer	Marks	Guidance
		$\Rightarrow u_{k+1} = 3u_k + 2k + 5 = 3\left(4(3)^k - k - 3\right) + 2k + 5$	M1	u_{k+1}, using u_k and attempting to simplify
		$= 4(3)^{k+1} - (k+1) - 3$	A1	Correct simplification or identification with a 'target' expression using $n = k+1$
		But this is the given result with $k+1$ replacing k. Therefore **if** it is true for $n = k$ **then** it is also true for $n = k+1$.	E1	The "target" shows this Dependent on A1 and previous E1
		Since it is true for $n = 1$, it is true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1
7	(i)	$\delta = 3 - 2\mathrm{j}$ and used	B1	
		$\sum$sum of roots = $\frac{9}{2}$ (correct sign in RHS)	M1	Or through obtaining two quadratic factors
		$\Rightarrow \alpha + \beta = \frac{9}{2} - (3+2\mathrm{j}) - (3-2\mathrm{j}) = -\frac{3}{2}$	A1	(AG)
		$\alpha\beta(3+2\mathrm{j})(3-2\mathrm{j}) = -13$ (correct sign in RHS)	M1	Or through obtaining two quadratic factors
		$\alpha\beta = -1$	A1 **[5]**	
7	(ii)	$\alpha\left(-\frac{3}{2} - \alpha\right) = -1 \Rightarrow 2\alpha^2 + 3\alpha - 2 = 0$	M1	Attempt at solution for α or β (from their $\alpha\beta$) Or from quadratic found earlier Or z^2 - (sum of roots)z + product of roots = 0
		$(2\alpha - 1)(\alpha + 2) = 0 \Rightarrow \alpha = \frac{1}{2}, -2$		
		$\alpha = \frac{1}{2}, \beta = -2$	A1	One root correct
			A1	Both roots correct (condone vice-versa)
			[3]	

Question		Answer	Marks	Guidance
7	(iii)	$\mathrm{f}(z) = (2z-1)(z+2)(z-(3+2\mathrm{j}))(z-(3-2\mathrm{j}))$ $= \left(2z^2+3z-2\right)\left(z^2-6z+13\right)$ $= 2z^4 - 9z^3 + 6z^2 + 51z - 26$	M1 A1 A1 **[3]**	A valid method seen to find A or B (by factors or root relations or substitution) May be seen earlier $A = 6$ $B = 51$ both cao
7	(iv)	$\mathrm{f}\left(\frac{w}{\mathrm{j}}\right) = 0 \Rightarrow w = \frac{1}{2}\mathrm{j},\ -2\mathrm{j},\ -2+3\mathrm{j},\ 2+3\mathrm{j}$	M1 A1 **[2]**	Their roots $\times\mathrm{j}$ FT
8	(i)	$x = 1$ $x = -4$ $y = 3$	B1 B1 B1 **[3]**	 SC " $x = 1, -4$ " is B1 B0
	(ii)	Evidence of method needed e.g. evaluating using 'large' values Large positive x, $y \to 3^-$ Large negative x, $y \to 3^+$	M1 A1 A1 **[3]**	 SC B1 for correct approaches without valid method seen

Question		Answer	Marks	Guidance
	(iii)		B1 B1 B1 **[3]**	3 branches correct relative to their asymptotes and carefully drawn Asymptotes correct and labelled Intercepts correct and labelled (allow 1.7 and -1.7)
	(iv)	$x < -4,\ -\sqrt{3} \le x < 1,\ x \ge \sqrt{3}$	B3 **[3]**	One mark for each. Correct inequality signs. Allow 1.73 for $\sqrt{3}$ (B3 then -1 if more than 3 inequalities)

Question		Answer	Marks	Guidance
9	(i)	$\sum_{r=1}^{n}\frac{2r+5}{(2r-1)(2r+1)(2r+3)}=\sum_{r=1}^{n}\left[\frac{3}{4(2r-1)}-\frac{1}{2r+1}+\frac{1}{4(2r+3)}\right]$	M1	Use of the given result (may be implied)
		$=\left(\frac{3}{4}-\frac{1}{3}+\frac{1}{20}\right)+\left(\frac{3}{12}-\frac{1}{5}+\frac{1}{28}\right)+\left(\frac{3}{20}-\frac{1}{7}+\frac{1}{36}\right)+\cdots$	M1 A1	Terms in full (first and at least one other) First term correct and one other correct
		$\cdots+\left(\frac{3}{4(2n-3)}-\frac{1}{2n-1}+\frac{1}{4(2n+1)}\right)+\left(\frac{3}{4(2n-1)}-\frac{1}{2n+1}+\frac{1}{4(2n+3)}\right)$	A1	Any 3 consecutive terms fully correct Or 2 consecutive algebraic terms fully correct (need not be simplified)
		$=\frac{3}{4}-\frac{1}{3}+\frac{1}{4}+\frac{1}{4(2n+1)}-\frac{1}{2n+1}+\frac{1}{4(2n+3)}$ $=\frac{2}{3}-\frac{3}{4(2n+1)}+\frac{1}{4(2n+3)}$ as required	M1	Valid attempt to cancel, including algebraic terms. Need to see the three algebraic fractions that remain after cancelling
			A1	Convincingly shown (AG)
			[6]	
9	(ii)	$\sum_{r=1}^{\infty}=\frac{2}{3}$	B1	
			[1]	
9	(iii)	$2r+5=45\Rightarrow r=20$	B1	May be implied (eg by using sum to 19 terms)
		$2r+5=105\Rightarrow r=50$	B1	May be implied
		$\sum_{r=1}^{50}-\sum_{r=1}^{19}=\left(\frac{2}{3}-\frac{3}{404}+\frac{1}{412}\right)-\left(\frac{2}{3}-\frac{1}{52}+\frac{1}{164}\right)$	M1	Difference of their sum from $r=1$ to 50 and their sum from $r=1$ to 19
		$=0.00813$	A1	Cao Invalid method A0 Unseen method SC B1
			[4]	

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4755 Further Concepts for Advanced Mathematics (FP1)

General Comments:

There were many good scripts from candidates who were well prepared for this paper.
Most candidates made very good progress through the first section. There was some evidence that many candidates were pressed for time and there were a surprising number of part questions not attempted, even when results that could be used were already given. Some of the algebraic manipulation required was quite lengthy, and for candidates lacking confidence in this area possibly too daunting.

There were several instances of the careless placing of answers, making it difficult for the Examiner to give the correct mark in the correct section. If a result has been obtained elsewhere there should be a signpost in the correct answer space, so that the Examiner can find the solution and reward it.

There was also much poor presentation, with work placed out of sequence and very often difficult to read. Figures should be rewritten, not altered. Use additional answer sheets when work is spilling out of the space available. Make sure that the annotations on a diagram are not obscured, and make sure that pencil diagrams are strongly drawn as the scanning process cannot pick up faint lines or shading.

Comments on Individual Questions:

Section A

Question No. 1
(i) This straight forward start to the paper was usually correct. Occasionally 8 – 2*p* was seen as the determinant. More rarely the determinant was forgotten, or there was an incorrect rearrangement of the elements of the matrix.

(ii) This was successfully done by the majority of candidates who multiplied the original area by the determinant. Some laboured methods to find the area of the original triangle by trigonometry usually produced an inaccurate answer. Some candidates worked with the transformed triangle; a decent sketch would have helped to find an easy method for the area in this case. Not many were successful.

Question No. 2
(i) Many correct results were seen here. Very rarely the meaning of z* was not known. The most common error was to mismanage the multiplication in the denominator, forgetting to square 5 or sometimes forgetting to use $j^2 = -1$. Another fairly common mistake was 10j in the numerator, instead of 20j.
(ii) Most candidates achieved full marks in this section, follow-through accuracy being allowed here. Errors in copying figures and sign errors in solving $2-b=\mathrm{Im}(z^*/z)$ were not uncommon.

Question No. 3
(i) The matrix row by column multiplications were usually correctly carried out. λ was usually correctly found, but an extremely common mistake was to obtain the value 25 from the matrix multiplication, but then to claim $\mu = \frac{1}{25}$.

(ii) The wrong μ in part (i) would then lose one of the marks in this part, where it would be incorrectly used to obtain the right inverse. To find $\mathbf{B}^{-1}$ the instruction in the question was "hence…"; direct work on **B** was not necessary and rarely successful.

Question No.4

(i) This part question was possibly the most successfully answered of all. There were those who were careless with brackets and produced ….+*p* in the final expression. A few candidates failed at the outset by not splitting the summation, attempting a multiplication $\sum r^2 \times \sum r$.

Candidates who were unwise enough to multiply out to a quartic expression could easily take out a factor of *n* but then had to demonstrate the factorisation of the cubic. The final result was given, so it was not enough to quote it at this stage, the factor $n+1$ needed justification.

(ii) It seemed that this section made algebraic demands beyond the abilities of many candidates. In trying to seek out the relevant terms by inspection many commonly failed to obtain both the ones in n^3. Several candidates thought that the problem could be solved by setting a value for *n*, sometimes zero, more often 1. A number did not attempt it at all.

Question No. 5

(i) This was well done by a majority of candidates. The circle was recognised to be a 'circle' but many 'circles' were extremely poorly drawn. Its placement was quite often centred on 3 – 4j. The half line was then also given a start point at 3 – 6j. The radius of the circle with either of these centres was such that the circle should have passed through the Origin. Quite a lot of candidates ignored this detail, and in the absence of a clear radius shown, only earned this mark if other identifiable points (e.g. -3 – j) were indicated on the diagram. Most candidates knew that a half line "straight up" was involved but sometimes annotations on the diagram were such that it appeared that the line started from the negative real axis, or from the circumference of the circle.

(ii) Given a correct diagram this was usually correct. Reverting to coordinates was penalised as the question specified that a complex number was required.

(iii) Most candidates, but not all, knew that the region required lay outside their circle, and most were able to indicate the other boundary and the region correctly. A fairly common mistake was to shade the inside of the circle between the two lines. Often the shading was difficult to make out on the screen after scanning. It should be strongly marked.

Question No. 6

An encouraging number of complete logical arguments were seen, but many candidates lost the final two marks for inadequate explanation. It is incorrect to claim the result is "true for *n* = *k* + 1" before both pointing out the structure and conditioning on the assumption. Using abbreviations such as "*n* = *k* + 1 is true" is also nonsensical and insufficient.

Section B

Question No.7

There were many routes possible to the solutions of the various parts to this question, but the examiners need to see the answers to each section in the right place and this was not always the case. Several candidates did the majority of the working for parts (i), (ii) and (iii) in part (i), by finding both quadratic factors, then using the sum and product of roots of the quadratic $2z^2+3z-2=0$ for part (i), solving the same quadratic for part (ii) and expanding the product of both quadratic factors to obtain the answers for part (iii). This was a very efficient method. In finding the product $\alpha\beta$ two common mistakes were to equate the product of the four roots to either $\frac{26}{2}$ or to -26. After this, quite a number of candidates gave up in (ii). Starting from the two correct equations for $\alpha+\beta$ and for $\alpha\beta$ some candidates were able to deduce the values of α and β by inspection.

Part (iv) was omitted by many and there were few correct answers. For the rest a common mistake was to divide the roots by j. It was evident that a fairly high proportion of candidates found the demands of this question too much to cope with and quite a number of scripts returned with no response in one or more parts.

Question No. 8

By contrast this question was probably the most successfully answered of all questions in the paper.
(i) Only a small number of candidates failed to write the asymptotes as the equations of lines. Three distinct equations are required. To write $x = 1, - 4$ is ambiguous. Rather than $x = 1$ or $x = - 4$ The asymptotes are both $x = 1$ and $x = - 4$.

(ii) The majority of candidates showed the calculations for large positive and negative values of x in full. The results are wanted to complete the justification and should not be omitted.

(iii) The curve was usually clearly and carefully drawn, with asymptotic behaviour usually well shown. Annotations are needed on the diagram to show the asymptotes' positions and the intercepts on the axes. These should not be left as working on the side, it is the sketch which is under scrutiny.

(iv) Most candidates scored all marks in this part, but there were some errors in the inequality signs used.

Question No. 9

There was evidence here from untidily rushed or incomplete work that candidates found that time was running out.

(i) Most candidates achieved the first four marks but there were quite a lot of errors, even in the initial terms. Sensible setting out of the work helped to generate the cancellation pattern of the fractions. Three consecutive values of r were needed to show this. The answer was given, and it was evident in many cases that the result was prematurely anticipated, either insufficient terms being shown or mistakes made which invalidated the work.

(ii) This question could be correctly answered from the given result to part (i), but even so, many wrong limits were chosen, the most popular being zero, and there were many omitted the part altogether.

(iii) Here there were many half-finished solutions and also no responses made. For the limits of the summation, 45 and 105 were often used. Using the result of (i) some candidates subtracted the sum from $r = 1$ to 20 instead of the sum from $r = 1$ to 19. In some cases premature approximation lost the final mark.

GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	57	52	47	42	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	49	42	35	29	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	64	57	51	45	39	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	59	53	48	43	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	60	54	48	43	38	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	60	54	49	44	39	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	67	61	55	49	43	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	59	53	47	41	36	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	60	53	46	40	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	60	54	48	42	37	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	48	43	38	34	30	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	55	50	45	40	36	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	55	49	44	39	33	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0

		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	48	43	38	34	30	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2016 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

Level 3 Certificate Mathematics for Engineering

			Max Mark	a*	a	b	c	d	e	u
H860	01 Mathematics for Engineering		This unit has no entries in June 2016							
H860	02 Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

			Max Mark	a*	a	b	c	d	e	u
H865	01 Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u
H866	01 Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H866	02 Critical maths	Raw	60	47	41	35	29	23	0
		Overall	132	111	96	81	66	51	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u
H867	01 Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H867	02 Statistical problem solving	Raw	60	40	34	28	23	18	0
		Overall	132	103	88	73	59	45	0

Advanced Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u
6993	01 Additional Mathematics	Raw	100	59	51	44	37	30	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u
6989	01 Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0

Version	Details of change
1.1	Correction to Overall grade boundaries for H866
	Correction to Overall grade boundaries for H867